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Wound String Scattering in NCOS Theory

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Abstract

We calculate the amplitude for a non-excited closed string with nonzero winding number to scatter from a D-string with a near critical E field. We go to the NCOS limit and observe that we get the same result if we adopt another approach put forward by Gomis and Ooguri.

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1 Introduction

There has recently been a lot of interest in the non-commutative open string (NCOS) theory living on a D-string [1, 2, 3]. The NCOS limit is reached by tuning the electric field along the D-string to its critical value which causes an infinite rescaling of the string tension and coupling. It is now understood that the NCOS limit is meaningful even in the absence of a D-brane, as demonstrated in the papers [4, 5]. This more general theory is called wound string theory [4], or non-relativistic closed string theory [5]. A number of calculations have been performed in the NCOS theory including various scattering amplitudes [2, 6, 7] and supergravity duals [8, 9]. As far as closed strings are concerned, NCOS theory has the usual rules for computing scattering amplitudes.

One can approach NCOS theory from a different direction using the set of rules proposed by [5]. These rules are obtained by taking a low energy limit already in the action, something which appears to precisely reproduce NCOS theory if a D-brane is present. In this sense the NCOS limit can be thought of as a non-relativistic limit.

It is certainly of value to confirm that these two different approaches give equivalent results. In this note we calculate the amplitude for a non-excited closed string to scatter from a D-string. In section 2 we use standard techniques for reaching the amplitude, taking the NCOS limit only afterwards. The strings have non-zero winding numbers and we will see that there are restrictions on the winding for the kinematic invariants to remain finite in the limit. The corresponding calculation with the new set of rules is shown in section 3, which, as expected, gives the same scattering amplitude. Finally, in section 4 we present some conclusions.

2 NCOS scattering

We calculate the amplitude for a non-excited NS-NS closed string with nonzero winding number to scatter from a D-string with a near critical E field. Similar calculations for unwound strings are performed in [10, 11] (without E -field) and [12] (with a near critical E -field). Taking the D-string along the x_1 direction, we split the metric according to

$$g_{ab} = \eta_{ab}, \quad g_{ij} = h\delta_{ij}, \quad a, b = 0, 1, \quad i, j = 2, \dots, 9, \quad (1)$$

and define the ‘image method’ matrix $D^\mu{}_\nu$ as [12],

$$D^a{}_b = \frac{1}{1-E^2} \begin{pmatrix} 1+E^2 & 2E \\ 2E & 1+E^2 \end{pmatrix}, \quad D^i{}_j = -\delta_{ij}. \quad (2)$$

Let $p_\mu = (k_0, k_1 + wR/\alpha', k)$ and $\tilde{p}_\mu = (k_0, k_1 - wR/\alpha', k)$ denote the momenta of the left- and right moving modes respectively, where w is the winding number around

the compact x_1 direction. For simplicity we choose $k_1 = n/R = 0$. We look at states polarized transversely to the D-string, i.e. each polarization tensor $\epsilon_{\mu\nu}$ obeys $\epsilon_{ab} = 0$. Additionally we require that $\epsilon^T = \epsilon$ and $\text{Tr}\epsilon = 0$. We may now write the scattering amplitude as [10, 12]

$$A \sim \frac{1}{\lambda} \sqrt{1 - E^2} \int d^2 z_1 d^2 z_2 \epsilon_{1\mu\nu} \epsilon_{2\sigma\kappa} \times \left\langle V_{-1}^\mu(p_{(1)}, z_1) V_{-1}^\nu(D \cdot \tilde{p}_{(1)}, \bar{z}_1) V_0^\sigma(p_{(2)}, z_2) V_0^\kappa(D \cdot \tilde{p}_{(2)}, \bar{z}_2) \right\rangle, \quad (3)$$

where the vertex operators are given by

$$\begin{aligned} V_{-1}^\mu(p, z) &= e^{-\phi(z)} \psi^\mu(z) e^{ip \cdot X(z)}, \\ V_0^\mu(p, z) &= (\partial X^\mu(z) + ip \cdot \psi(z) \psi^\mu(z)) e^{ip \cdot X(z)}. \end{aligned} \quad (4)$$

In (3) we used the fact that for transverse polarizations we get $\epsilon \cdot D = D \cdot \epsilon = -\epsilon$. Keeping track of the different modes it is seen that the results can be inferred from [10, 12] by simply replacing $D \cdot p$ with $D \cdot \tilde{p}$. Momentum conservation in the directions parallel to the D-string requires that

$$p_{(1)} + D \cdot \tilde{p}_{(1)} + p_{(2)} + D \cdot \tilde{p}_{(2)} = 0, \quad (5)$$

which implies

$$\begin{aligned} p_{(1)} \cdot p_{(2)} &= \tilde{p}_{(1)} \cdot \tilde{p}_{(2)} = 2t/\alpha', & p_{(1)} \cdot D \cdot \tilde{p}_{(2)} &= \tilde{p}_{(1)} \cdot D \cdot p_{(2)} \\ \text{and} & & p_{(1)} \cdot D \cdot \tilde{p}_{(1)} &= p_{(2)} \cdot D \cdot \tilde{p}_{(2)} = 2s/\alpha', \end{aligned} \quad (6)$$

where we have defined the kinetic invariants s and t . Multiplying (5) by ϵ_i we see that $p_{(i)} \cdot \epsilon_j = \tilde{p}_{(i)} \cdot \epsilon_j$. Using these relations, all but one of the kinematic factors vanish or cancel out, and what remains is identical to the corresponding result in [10] except for a change $p_{(2)} \cdot D \cdot p_{(2)} \text{Tr}(\epsilon_1 \cdot \epsilon_2) \rightarrow p_{(2)} \cdot D \cdot \tilde{p}_{(2)} \text{Tr}(\epsilon_1 \cdot \epsilon_2)$. Below we show a few steps in the calculation. Eliminating terms which eventually add up to zero, we can write

$$A \sim \frac{1}{\lambda} \sqrt{1 - E^2} \int d^2 z_1 d^2 z_2 C(z_1, \bar{z}_1, z_2, \bar{z}_2) \times \left\langle : e^{ip_{(1)} \cdot X(z_1)} :: e^{iD \cdot \tilde{p}_{(1)} \cdot X(\bar{z}_1)} :: e^{ip_{(2)} \cdot X(z_2)} :: e^{iD \cdot \tilde{p}_{(2)} \cdot X(\bar{z}_2)} : \right\rangle, \quad (7)$$

where $C(z_1, \bar{z}_1, z_2, \bar{z}_2)$ contains the contractions not involving the $X^\mu(z)$'s. Fixing the $SL(2, R)$ invariance by putting $z_1 = i$ and $z_2 = iy$ we get the Jacobian $d^2 z_1 d^2 z_2 \rightarrow 4(1 - y^2)dy$ and, for the contractions

$$C(i, -i, iy, -iy) = \frac{1}{(1 - y^2)^2} \text{Tr}(\epsilon_1 \cdot \epsilon_2) s \quad (8)$$

and

$$\begin{aligned} \left\langle : e^{ip_{(1)} \cdot X(i)} :: e^{iD \cdot \tilde{p}_{(1)} \cdot X(-i)} :: e^{ip_{(2)} \cdot X(iy)} :: e^{iD \cdot \tilde{p}_{(2)} \cdot X(-iy)} : \right\rangle &= \\ &= (-1)^s 2^s (2y)^s (1 - y)^{2t} (1 + y)^{-2s - 2t}. \end{aligned} \quad (9)$$

A change of variables to $y = (1 - \sqrt{x})/(1 + \sqrt{x})$ gives a standard Euler beta integral, and we get

$$A = \frac{1}{\lambda} \sqrt{1 - E^2} \frac{\Gamma(t)\Gamma(s+1)}{\Gamma(1+s+t)} \text{Tr}(\epsilon_1 \cdot \epsilon_2) s. \quad (10)$$

Writing out s explicitly we have

$$s = -\frac{1 + E^2}{1 - E^2} \frac{\alpha'}{2} (k_0)^2 - \frac{1 + E^2}{1 - E^2} \frac{\alpha'}{2} \left(\frac{wR}{\alpha'} \right)^2 - \frac{\alpha'}{2h} k^2 - \frac{2EwR}{1 - E^2} k_0. \quad (11)$$

Notice that the mass-shell condition (for arbitrary closed string states) implies that

$$k_0 = \sqrt{\left(\frac{wR}{\alpha'} \right)^2 + \left(\frac{n}{R} \right)^2 + \frac{k^2}{h} + \frac{2}{\alpha'} (N + \tilde{N})}. \quad (12)$$

We will be looking at the special case where $N = \tilde{N} = n = 0$. Using (12) to eliminate k_0 in (11), we conclude that

$$s = -\frac{\alpha'}{1 - E^2} \left(\frac{EwR}{\alpha'} + \sqrt{\left(\frac{wR}{\alpha'} \right)^2 + \frac{k^2}{h}} \right)^2. \quad (13)$$

We now take the NCOS limit $\varepsilon \rightarrow 0$ with $E = 1 - \varepsilon/2$, $\alpha' = \varepsilon \alpha'_e$ and $h = \varepsilon$. For $w > 0$, which is needed for s to remain finite in the limit, we get as $\varepsilon \rightarrow 0$

$$s = -\alpha'_e \left(\frac{wR}{2\alpha'_e} + \frac{\alpha'_e k^2}{2wR} \right)^2 = -\alpha'_e (k_0^{\text{NCOS}})^2, \quad (14)$$

where we have introduced k_0^{NCOS} as an abbreviation. Note that in the absence of an electric field we can write the “image method” matrix as $D = V - N$ where V and N projects out parallel and perpendicular (to the brane) parts respectively so that

$$s = \frac{\alpha'}{2} (pD\tilde{p}) = \frac{\alpha'}{4} (p + D\tilde{p})^2 = \frac{\alpha'}{4} (V(p + \tilde{p}))^2 = \alpha' k_0^2 g^{00}. \quad (15)$$

The net effect of the (near critical) electric field is in the present case $\alpha' g^{\mu\nu} \rightarrow \alpha'_e \eta^{\mu\nu}$. Taking the limit amounts to replace k_0 with k_0^{NCOS} . Analogously, for t we have

$$t = \frac{\alpha'_e}{4} (k_{(1)} + k_{(2)} + 2k_{(1)}k_{(2)}), \quad (16)$$

which is just $\alpha' g^{\mu\nu} \rightarrow \alpha'_e \eta^{\mu\nu}$ applied to

$$\begin{aligned} t &= \frac{\alpha'}{4} (p_{(1)} + p_{(2)})^2 = \frac{\alpha'}{4} (V(p_{(1)} + p_{(2)}) + N(p_{(1)} + p_{(2)}))^2 \\ &= \frac{\alpha'}{4} (k_{(1)} + k_{(2)})_i g^{ij} (k_{(1)} + k_{(2)})_j. \end{aligned} \quad (17)$$

3 Wound string theory calculation

It is interesting to check whether we arrive at the same result in wound string theory using the set of rules proposed by [5]. These rules are obtained by taking the critical limit already in the action. In this limit, the bosonic part of the world sheet action is [5]

$$S = \int \frac{d^2z}{2\pi} \left(\beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} + \frac{1}{4\alpha'_e} \partial \gamma \bar{\partial} \bar{\gamma} + \frac{1}{\alpha'_e} \partial X^i \bar{\partial} X_i \right), \quad (18)$$

with $\gamma = X^0 + X^1$, $\bar{\gamma} = -X^0 + X^1$ and the β 's as Lagrange multipliers. The zero mode parts of the momenta conjugate to γ and $\bar{\gamma}$ are

$$\frac{1}{2}(k_0 + k_1) = i\beta_0 + \frac{wR}{4\alpha'_e} \quad \text{and} \quad \frac{1}{2}(k_0 - k_1) = i\bar{\beta}_0 + \frac{wR}{4\alpha'_e} \quad (19)$$

and the Virasoro constraint is

$$i\beta_0 = \frac{N}{wR} + \frac{\alpha'_e k^2}{4wR}, \quad i\bar{\beta}_0 = \frac{\tilde{N}}{wR} + \frac{\alpha'_e k^2}{4wR}. \quad (20)$$

Thus the energy in wound string theory, k_0^W , is given by

$$k_0^W = v + \bar{v} + \frac{wR}{2\alpha'_e} = \frac{wR}{2\alpha'_e} + \frac{\alpha'_e k}{2wR} + \frac{N + \tilde{N}}{2wR}, \quad (21)$$

where v and \bar{v} are the eigenvalues of zero mode $i\bar{\beta}$ and $i\beta$ respectively. Note that $k_0^W = k_0^{\text{NCOS}}$. The momentum k_1 conjugate to the compact direction x_1 is given by

$$k_1 = \bar{v} - v. \quad (22)$$

For $N + \tilde{N} = 0$ and $k_1 = 0$ we get

$$v = \bar{v} = \frac{\alpha'_e k}{4wR}. \quad (23)$$

We would now like to check that the scattering amplitude calculated in this framework agrees with the NCOS amplitude in the previous section. To do this, we make use of the propagators below, where $\mu, \nu \geq 2$,

$$\begin{aligned} \langle X^\mu(z_i) X^\nu(z_j) \rangle &= -\frac{\alpha'_e}{2} \eta^{\mu\nu} \log(z_i - z_j), & \langle X^\mu(z_i) \tilde{X}^\nu(\bar{z}_j) \rangle &= \frac{\alpha'_e}{2} \eta^{\mu\nu} \log(z_i - \bar{z}_j), \\ \langle \bar{\gamma}(\bar{z}_i) \int^{\bar{z}_j} \bar{\beta} \rangle &= \log(\bar{z}_i - \bar{z}_j), & \langle \gamma(z_i) \int^{z_j} \beta \rangle &= \log(z_i - z_j), \\ \langle \gamma(z_i) \bar{\gamma}(\bar{z}_j) \rangle &= 4\alpha'_e \log(z_i - \bar{z}_j), & \langle \int^{z_i} \beta \int^{\bar{z}_j} \bar{\beta} \rangle &= \frac{1}{4\alpha'_e} \log(z_i - \bar{z}_j). \end{aligned} \quad (24)$$

The interesting part of the amplitude, $\langle \Pi : e^{ip(i)X} : \rangle$, is now given by

$$\begin{aligned}
& \langle : e^{iv_{(1)}\gamma(z_1)+iw_{(1)}R\int^{z_1}\beta+ik_{(1)}X(z_1)} :: e^{iv_{(1)}\bar{\gamma}(\bar{z}_1)+iw_{(1)}R\int^{\bar{z}_1}\bar{\beta}+ik_{(1)}\tilde{X}(\bar{z}_1)} : \\
& \quad : e^{iv_{(2)}\gamma(z_2)+iw_{(2)}R\int^{z_2}\beta+ik_{(2)}X(z_2)} :: e^{iv_{(2)}\bar{\gamma}(\bar{z}_2)+iw_{(2)}R\int^{\bar{z}_2}\bar{\beta}+ik_{(2)}\tilde{X}(\bar{z}_2)} : \rangle = \\
& = (z_1 - \bar{z}_1)^{-\frac{\alpha'_e}{4}(\frac{wR}{\alpha'_e}+4v)^2} (z_2 - \bar{z}_2)^{-\frac{\alpha'_e}{4}(\frac{wR}{\alpha'_e}+4v)^2} \\
& \quad \times |z_1 - z_2|^{4vwR+\alpha'_e k_{(1)}k_{(2)}} |z_1 - \bar{z}_2|^{\frac{\alpha'_e}{2}(\frac{wR}{\alpha'_e}+4v)^2-4vwR-\alpha'_e k_{(1)}k_{(2)}} \\
& = (z_1 - \bar{z}_1)^s (z_2 - \bar{z}_2)^s |z_1 - z_2|^{2t} |z_1 - \bar{z}_2|^{-2s-2t} \\
& = (-1)^s 2^s (2y)^s (1-y)^{2t} (1+y)^{-2t-2s}. \tag{25}
\end{aligned}$$

In the second equality we have used that s and t can be written as

$$\begin{aligned}
s &= -\frac{\alpha'_e}{4} \left(\frac{wR}{\alpha'_e} + 4v \right)^2 \\
t &= -v_{(1)}w_{(2)}R - v_{(2)}w_{(1)}R + \frac{\alpha'_e}{2}k_{(1)}k_{(2)} = 2vwR + \frac{\alpha'_e}{2}k_{(1)}k_{(2)},
\end{aligned} \tag{26}$$

where $w = w_{(1)} = -w_{(2)}$ and $v = v_{(1)} = -v_{(2)}$. As expected, we recover the NCOS amplitude as far as the bosonic fields are concerned. The fermionic fields can be treated analogously, adding up to an amplitude which is identical to that in the previous section.

4 Conclusions

In this paper we have calculated the amplitude for a non-excited closed string to scatter from a near critical D-string. We allow for nonzero winding numbers so that the kinetic invariants can remain finite in the limit. Two different approaches to the calculation were presented. We started off by using standard rules, waiting until the end before taking the NCOS limit. Then we adopted the set of rules put forward in [5], which amounts to taking a proper limit already in the action. We have thus been able to confirm that the two approaches give rise to equivalent amplitudes.

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